Syllabus for Q Exam on Probability and Random Processes


Basic concepts: sample spaces, probability measures, outcomes, events, combinatorial approaches to computing probabilities, conditioning, total probability, independence, Bayes’ rule;

Random variables: definition of, probability mass functions (PMFs), probability density functions (PDFs), cumulative distribution functions (CDFs), commonly-used distributions, expectations, characteristic functions, moment inequalities;

Random vectors: definition of, joint PMFs, PDFs, and CDFs, joint characteristic functions, conditional distributions and conditional expectation, joint moments, covariance matrices and their properties, jointly Gaussian random variables;

Limit theorems: law of large numbers, central limit theorem

Estimation: LLSE and MMSE estimators

Detection: MAP and ML detectors

Second-order random processes: stationarity and wide-sense stationarity, autocorrelation, power spectral density, white noise, filtered random processes;

Discrete-time Markov chains: definition, conditions for stationarity, $n$-step transition probabilities, stationary distributions, occupancy rates;

Continuous-time Markov chains: definition, conditions for stationarity, the forward and backward equations, Poisson processes, the M/M/1 queue, occupancy rates.
Syllabus for Q-exam on Linear Algebra, Signals, and Systems

References: linear algebra at the level of Gilbert Strang’s *Introduction to Linear Algebra* (see also the MIT courseware http://web.mit.edu/18.06/www/) or Sheldon Axler’s *Linear Algebra Done Right*. Signals and systems concepts at the level of A. V. Oppenheim and A. S Wilsky’s *Signals and Systems*.

**Linear algebra:** vector spaces, linear mappings, spanning sets, bases and dimension of finite-dimensional vector spaces; nullspace, range, and rank of arbitrary real and complex matrices; determinant, trace, invertibility, eigenvalues, and eigenvectors of square real and complex matrices; inner-product spaces and orthogonal/unitary diagonalizability of Hermitian matrices; singular-value decomposition of arbitrary real and complex matrices; condition number of invertible square matrices.

**Signals basics:** real- and complex-valued continuous- and discrete-time signals; convolution in continuous and discrete time.

**Systems basics:** single-input single-output LTI systems in continuous and discrete time; impulse response; causality and BIBO stability of SISO LTI systems (definitions and impulse-response criteria).

**Spectral concepts in continuous time:** Fourier series of continuous-time periodic signals; Fourier transforms of continuous-time signals; the idea of frequency content and bandwidth of continuous-time signals; frequency response of continuous-time LTI systems; ideal filters.

**Spectral concepts in discrete time:** the discrete-time Fourier transform and the Sampling Theorem; frequency response of discrete-time LTI systems; The DFT and the FFT for N-point signals.

**Other transforms and applications:** the two-sided $z$-transform and two-sided Laplace transform; transfer functions of continuous- and discrete-time SISO LTI systems; criteria for BIBO stability in terms of transfer functions.